

Dalitz plot analysis and branching fraction measurement of D^+ and $D_s^+ \rightarrow \pi^- \pi^+ \pi^+$ decays *

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Fermilab fixed target experiment E791 obtained a sample of 1172 ± 61 events of $D^+ \rightarrow \pi^- \pi^+ \pi^+$ and 848 ± 44 events of $D_s^+ \rightarrow \pi^- \pi^+ \pi^+$ decays. We find respectively $B(D^+ \rightarrow \pi^- \pi^+ \pi^+)/B(D^+ \rightarrow K^- \pi^+ \pi^+) = 0.0311 \pm 0.0018^{+0.0016}_{-0.0026}$ and $B(D_s^+ \rightarrow \pi^+ \pi^- \pi^+)/B(D_s^+ \rightarrow \phi \pi^+) = 0.245 \pm 0.028^{+0.019}_{-0.012}$. Using a coherent amplitude analysis to fit the Dalitz plot of the $D^+ \rightarrow \pi^- \pi^+ \pi^+$ decay, we find strong evidence for a scalar resonance of mass $478^{+24}_{-23} \pm 17$ MeV/ c^2 and width $324^{+42}_{-40} \pm 21$ MeV/ c^2 , compatible with what is expected for the isoscalar meson σ . The $D^+ \rightarrow \sigma(500)\pi^+$ fraction accounts for approximately half of all three-charged-pion decays of the D^+ . From the Dalitz plot analysis of the $D_s^+ \rightarrow \pi^- \pi^+ \pi^+$ decay events, we find significant contributions from the channels $\rho^0(770)\pi^+$, $\rho^0(1450)\pi^+$, $f_0(980)\pi^+$, $f_2(1270)\pi^+$, and $f_0(1370)\pi^+$. We also present new measurement of the masses and widths of the isoscalar resonances $f_0(980)$ and $f_0(1370)$.

1. Introduction^a

E791 data were produced by 500 GeV/ c π^- interactions in five thin foils (one platinum, four diamond) separated by gaps of 1.34 to 1.39 cm. The detector, the data set, the reconstruction, and the resulting vertex resolutions have been described previously [1]. After reconstruction, events with evidence of well-separated production (primary) and decay (secondary) vertices were retained for further analysis. From the 3-prong secondary vertex candidates, we select a $\pi^- \pi^+ \pi^+$ sample with invariant mass ranging from 1.7 to 2.1 GeV/ c^2 . For this analysis all charged particles are taken to be pions; i.e., no direct use is made of particle identification.

2. D^+ and $D_s^+ \rightarrow \pi^- \pi^+ \pi^+$ relative Branching Ratio

We fit the spectrum of Figure 1 as the sum of D^+ and D_s^+ signals plus background. We model the background as the sum of four components: a general combinatorial background, the reflection of the $D^+ \rightarrow K^- \pi^+ \pi^+$ decay, reflections of $D^0 \rightarrow K^- \pi^+$ plus one extra track (mostly from the primary vertex), and $D_s^+ \rightarrow \eta' \pi^+$ followed by $\eta' \rightarrow \rho^0(770)\gamma$, $\rho^0(770) \rightarrow \pi^+ \pi^-$. We use Monte Carlo (MC) simulations to determine the shape of each identified charm background in the $\pi^- \pi^+ \pi^+$ spectrum [2]. We assume that the combinatorial background falls exponentially with mass. The fit finds 1172 ± 61 D^+ events and 848 ± 44 D_s^+ events.

The branching fraction for $D_s^+ \rightarrow \pi^- \pi^+ \pi^+$ relative to that for $D_s^+ \rightarrow \phi \pi^+$ is measured to be:

$$\frac{B(D_s^+ \rightarrow \pi^- \pi^+ \pi^+)}{B(D_s^+ \rightarrow \phi \pi^+)} = 0.245 \pm 0.028^{+0.019}_{-0.012} . \quad (1)$$

The branching fraction for $D^+ \rightarrow \pi^- \pi^+ \pi^+$ relative to that of $D^+ \rightarrow K^- \pi^+ \pi^+$ is measured to be:

$$\frac{B(D^+ \rightarrow \pi^- \pi^+ \pi^+)}{B(D^+ \rightarrow K^- \pi^+ \pi^+)} = 0.0311 \pm 0.0018^{+0.0016}_{-0.0026} . \quad (2)$$

The first error is statistical and the second is systematic.

3. Dalitz plot formalism

To study the resonant structure of these decays we consider the 1686 events with invariant mass

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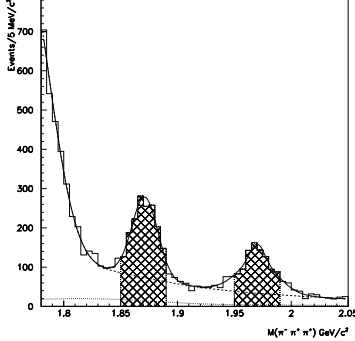


Figure 1. The $\pi^-\pi^+\pi^+$ effective mass spectrum. The dotted line represents the $D^0 \rightarrow K^-\pi^+$ plus $D_s^+ \rightarrow \eta'\pi^+$ and the dashed line is the total background. Events in the hatched areas at the D_s and D^+ mass are used for the D_s and D^+ Dalitz plot analyses.

between 1.85 and 1.89 GeV, for the D^+ analysis and the 937 events with invariant mass between 1.95 and 1.99 GeV/ c^2 for the D_s^+ . Fig. 2a,b shows the Dalitz plot for these events. The horizontal and vertical axes are the squares of the $\pi^+\pi^-$ invariant masses, and the plot has been symmetrized with respect to the two π^+ 's.

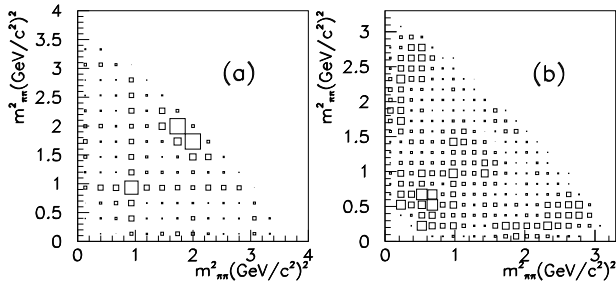


Figure 2. (a) The $D_s^+ \rightarrow \pi^-\pi^+\pi^+$ Dalitz plot and (b) the $D^+ \rightarrow \pi^-\pi^+\pi^+$ Dalitz plot. Since there are two identical particles, the plots are symmetrized.

We fit each distribution shown in Figure 2 to a signal probability distribution function (PDF), which is a coherent sum of amplitudes corresponding to the non-resonant decay plus five different resonant channels, and a background

PDF of known shape and magnitude. The resonant channels we include in the fit are $\rho^0(770)\pi^+$, $f_0(980)\pi^+$, $f_2(1270)\pi^+$, $f_0(1370)\pi^+$, and $\rho^0(1450)\pi^+$.

We assume the non-resonant amplitude to be uniform across the Dalitz plot. Each resonant amplitude, except that for the $f_0(980)$, is parameterized as a product of form factors, a relativistic Breit-Wigner function, and an angular momentum amplitude which depends on the spin of the resonance [3].

For the $f_0(980)\pi^+$ we use a coupled-channel Breit-Wigner function, following the parameterization of the WA76 Collaboration [4],

$$BW_{f_0(980)} = \frac{1}{m_{\pi\pi}^2 - m_0^2 + im_0(\Gamma_\pi + \Gamma_K)}, \quad (3)$$

with

$$\Gamma_\pi = g_\pi \sqrt{m_{\pi\pi}^2/4 - m_\pi^2} \quad (4)$$

and

$$\Gamma_K = \frac{g_K}{2} \left(\sqrt{m_{\pi\pi}^2/4 - m_{K^+}^2} + \sqrt{m_{\pi\pi}^2/4 - m_{K^0}^2} \right). \quad (5)$$

We multiply each amplitude by a complex coefficient, $c_j = a_j e^{i\delta_j}$. The fit parameters are the magnitudes, a_j , and the phases, δ_j , which accommodate the final state interactions. The reported parameters values are obtained using the maximum-likelihood method.

4. $D_s^+ \rightarrow \pi^-\pi^+\pi^+$ Dalitz plot results

The parameters of the $f_0(980)$ state, g_π , g_K , and m_0 , as well as the mass and width of the $f_0(1370)$, are determined directly from the $D_s^+ \rightarrow \pi^-\pi^+\pi^+$ decay events, floating them as free parameters in the fit. The measured values $m_0 = 977 \pm 3 \pm 2$ MeV/ c^2 , $g_\pi = 0.09 \pm 0.01 \pm 0.01$ and $g_K = 0.02 \pm 0.04 \pm 0.03$. For the $f_0(1370)$ we find $m_0 = 1434 \pm 18 \pm 9$ MeV/ c^2 and $\Gamma_0 = 173 \pm 32 \pm 6$ MeV/ c^2 . In both results, the first reported error is statistical and the second is systematic. The other resonance masses and widths are taken from the PDG [5].

Table 1 shows the phases (δ_j) determined from the fit, and corresponding fraction for each decay mode. We calculate the decay fraction for each

mode	relative phase	fraction(%)
$f_0(980)\pi^+$	0° (fixed)	$57. \pm 4. \pm 5$
non-resonant	$(181. \pm 94. \pm 51.)^\circ$	$.5 \pm 1. \pm 2.$
$\rho^0(770)\pi^+$	$(109. \pm 24 \pm 5.)^\circ$	$6. \pm 2. \pm 4$
$f_2(1270)\pi^+$	$(133. \pm 13. \pm 28.)^\circ$	$20. \pm 3. \pm 1.$
$f_0(1370)\pi^+$	$(198. \pm 19. \pm 27.)^\circ$	$32. \pm 8 \pm 2.$
$\rho^0(1450)\pi^+$	$(162. \pm 26. \pm 17.)^\circ$	$4. \pm 2. \pm .2$

Table 1. Results of the D_s^+ Dalitz plot fits (systematic error follows the statistical).

amplitude as its intensity, integrated over the Dalitz plot, divided by the integrated intensity of the signal's coherently summed amplitudes. The first reported error is statistical and the second is systematic, the latter being dominated by the uncertainties in the resonance parameters, in the background parameterization, and in the acceptance correction. The $f_0(980)\pi^+$ is the dominant component, accounting for nearly half of the $D_s^+ \rightarrow \pi^-\pi^+\pi^+$ decay width, followed by the $f_0(1370)\pi^+$ and $f_2(1270)\pi^+$ components. The contribution of $\rho^0(770)\pi^+$ and $\rho^0(1450)\pi^+$ components corresponds to about 10% of the $\pi^-\pi^+\pi^+$ width. We have not found a statistically significant non-resonant component. The two $m_{\pi^+\pi^-}^2$ projections are nearly independent and the sum of them is shown in Fig. 3.

To assess the quality of our fit absolutely, and to compare it with other possible fits, we developed a fast-MC algorithm that simulates the $D_s^+ \rightarrow \pi^-\pi^+\pi^+$ Dalitz plot from a given signal distribution, background, detector resolution and acceptance. For any given set of input parameters we calculated a χ^2 using the procedure presented in Ref. [3]. From χ^2 and the number of degrees of freedom (ν), we calculate a confidence level assuming a Gaussian distribution in χ^2/ν . The confidence level for the agreement of the projection of our result onto the Dalitz plot with the data of table I is 35%.

5. $D^+ \rightarrow \pi^-\pi^+\pi^+$ Dalitz plot analysis results

In a first model for this channel, which we will refer to as Fit 1, the signal PDF includes the same resonances states used in the $D_s^+ \rightarrow \pi^-\pi^+\pi^+$ analysis. In the case of the $f_0(980)$ and $f_0(1370)$, we used the parameters obtained in the $D^+ \rightarrow \pi^-\pi^+\pi^+$ analysis. In this model, the non-

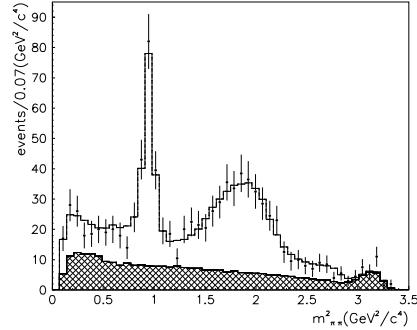


Figure 3. $m_{\pi\pi}^2$ projections for data (dots) and fast-MC (solid). The hashed area is the background distribution.

resonant, the $\rho^0(1450)\pi^+$ and the $\rho^0(770)\pi^+$ amplitudes dominate [3]. The qualitative features of this fit are similar to those reported by E691[6] and E687[7].

We produce the χ^2 distribution for the difference in densities and observe a concentration of χ^2 in the low $\pi^+\pi^-$ mass² ($m_{\pi^+\pi^-}^2$) region. The χ^2 summed over all bins is 254 for 162 degrees of freedom (ν), which corresponds to a confidence level less than 10^{-5} , assuming Gaussian errors. We display the sum of the two $m_{\pi\pi}^2$ in Fig. 4a for the data and for the fast-MC.

The low confidence level casts doubt on the validity of the model used. While the projection of the MC onto the $\pi^+\pi^-$ mass² axis describes the data in the $\rho^0(770)$ and $f_0(980)$ regions well, there is a discrepancy at lower mass, suggesting the possibility of another amplitude. To investigate the possibility that another $\pi^+\pi^-$ resonance contributes an amplitude to the $D^+ \rightarrow \pi^-\pi^+\pi^+$ decay, we add a sixth resonant amplitude to the signal PDF. We allow its mass and width to float and assume a scalar angular distribution. This fit (Fit 2) converges and finds values of $478^{+24}_{-23} \pm 17$ MeV/ c^2 for the mass and $324^{+42}_{-40} \pm 21$ MeV/ c^2 for the width, the first error is statistical and the second systematic. We will refer to this possible state as the $\sigma(500)$.

In Fit 2, the $\sigma(500)$ amplitude produces the largest decay fraction, 46%, with a relatively small statistical error, 9%. The non-resonant fraction, which at $(39 \pm 10)\%$ was the largest in

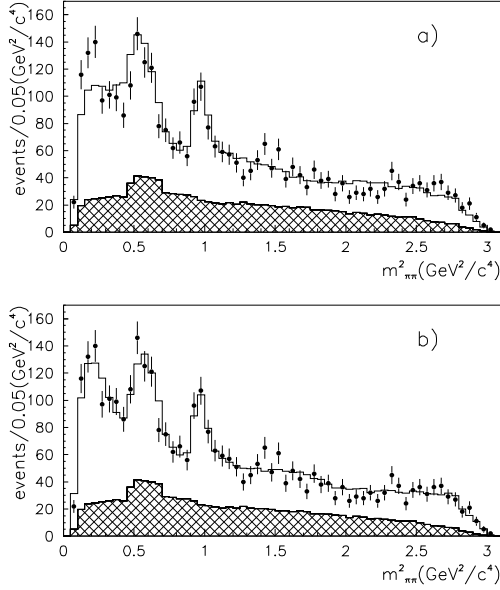


Figure 4. $m^2_{\pi\pi}$ projection for data (error bars) and fast-MC (solid). The shaded area is the background distribution, (a) solution with the Fit 1, (b) solution with Fit 2.

the original fit, is now only $(8 \pm 6)\%$. When we project this model onto the Dalitz plot, the χ^2/ν becomes 138/162. The projection of this model onto the $\pi^+\pi^-$ invariant mass squared distribution, shown in Fig. 4b, describes the data well, including the accumulation of events near $0.2 \text{ GeV}^2/c^4$.

mode	relative phase	fraction(%)
$\sigma\pi^+$	$(207. \pm 8. \pm 5.)^\circ$	$46. \pm 9. \pm 2.$
$\rho^0(770)\pi^+$	$0^\circ(\text{fixed})$	$34. \pm 3. \pm 2.$
non-resonant	$(57. \pm 20. \pm 6.)^\circ$	$8. \pm 6. \pm 3.$
$f_0(980)\pi^+$	$(165. \pm 11. \pm 3.)^\circ$	$6. \pm 1. \pm .4$
$f_2(1270)\pi^+$	$(57. \pm 8. \pm 3.)^\circ$	$19. \pm 3. \pm 0.4$
$f_0(1370)\pi^+$	$(105. \pm 18. \pm 1.)^\circ$	$2.5 \pm 1.5 \pm 1.$
$\rho^0(1450)\pi^+$	$(319. \pm 29. \pm 11.)^\circ$	$1. \pm 1. \pm .3$

Table 2. Results of the D^+ Dalitz plot fit, including the scalar $\sigma(500)$ resonance. (the systematic error follows the statistical).

To better understand our data, we also fit it with vector, tensor, and toy models for the sixth

(sigma) amplitude, allowing the masses, widths, and relative amplitudes to float freely. The vector and tensor models test the angular distribution of the signal. The toy model tests the phase variation expected of a Breit-Wigner amplitude by substituting a constant relative phase. These alternative explanations of the data fail to describe it as well the scalar with a regular Breit Wigner option [3].

6. Conclusions

Using a Dalitz plot analysis of the three pion decay of the D_s^+ , we find a little but significant contributions from the channels $\rho^0(770)\pi^+$, $\rho^0(1450)\pi^+$. We measure the mass and width of the $f_0(980)$ and we have not found a statistically significant g_K component in the width. Using the same analysis for the D^+ mass region, we find strong evidence that a scalar resonance with mass $478^{+24}_{-23} \pm 17 \text{ MeV}/c^2$ and width $324^{+42}_{-40} \pm 21 \text{ MeV}/c^2$ produces a decay fraction $\approx 50\%$. These results are discussed in references [2, 3, 8].

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